Black Holes and Thermodynamics

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S. Chandrasekhar and Black Holes

S. Chandraskekar (1975): *In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity, discovered by the New Zealand mathematician, Roy Kerr, provides the absolutely exact representation of untold numbers of massive black holes that populate the universe. This shuddering before the beautiful, this incredible fact that a discovery motivated by a search after the beautiful in mathematics should find its exact replica in Nature, persuades me to say that beauty is that to which the human mind responds at its deepest and most profound.*

This quote illustrates Chandra's view of black holes: Mathematically, they are simple and elegant objects, but they describe nature in a deep and profound way.

I believe that the relationship between black holes and thermodynamics provides us with the deepest insights that we currenly have concerning the nature of gravitation, thermodynamics, and quantum physics.

Although Chandra himself did not work directly on black hole thermodynamics, I believe that this topic is a highly appropriate one for a symposium in his honor.

Black Holes

Black Holes: A *black hole* is a region of spacetime where gravity is so strong that nothing—not even light—that enters that region can ever escape from it.

Michell (1784); Laplace (1798):

Escape velocity:

$$\frac{1}{2}mv_e^2 = G\frac{mM}{R}$$

so $v_e > c$ if

$$R < R_S \equiv \frac{2GM}{c^2} \approx 3\frac{M}{M_\odot}\mathrm{km}$$

Michell and Laplace predicted that stars with $R < R_S$ would appear to be black.

General relativity Nothing can travel faster than light, so if light is "pulled back", then so is everything else. A body with $R < \frac{2GM}{c^2}$ cannot exist in equilibrium; it must undergo complete gravitational collapse to a singularity. There is considerable (but mainly indirect!) evidence in favor of the "cosmic censorship conjecture": The end product of this collapse is always a black hole, with the singularity hidden within the black hole.

Formation of Black Holes in Astrophysics

1. Stellar collapse: After exhaustion of its thermonuclear fuel, a star can support itself against collapse under its own weight only if it is able to generate pressure without high temperature:

- For $M < 1.4 M_\odot$ support by electron degeneracy pressure is possible: white dwarfs.

  S. Chandrasekhar (1934): *A star of large mass cannot pass into the white-dwarf stage and one is left speculating on other possibilities.*

- For $M <\sim 2 M_\odot$ support by neutron degeneracy pressure/nuclear forces is possible: neutron stars

- However, if $M >\sim 2M_\odot$ and the excess mass is not shed (e.g., in a supernova explosion), complete gravitational collapse is unavoidable: black holes

Mass range of black holes formed by stellar collapse: $\sim 2M_\odot < M <\sim 100M_\odot$. About 20 very strong candidates are known from binary X-ray systems.

2. Collapse of the central part of a galactic nucleus or star cluster: A variety of processes can plausibly lead to the formation of massive black holes at the center of galactic nuclei or dense clusters of stars. Black holes are believed to be the "central engine" of quasars. Massive black holes are believed to be present at the centers of most galaxies.

Mass range: $\sim 10^5 M_\odot < M <\sim 10^{10} M_\odot$

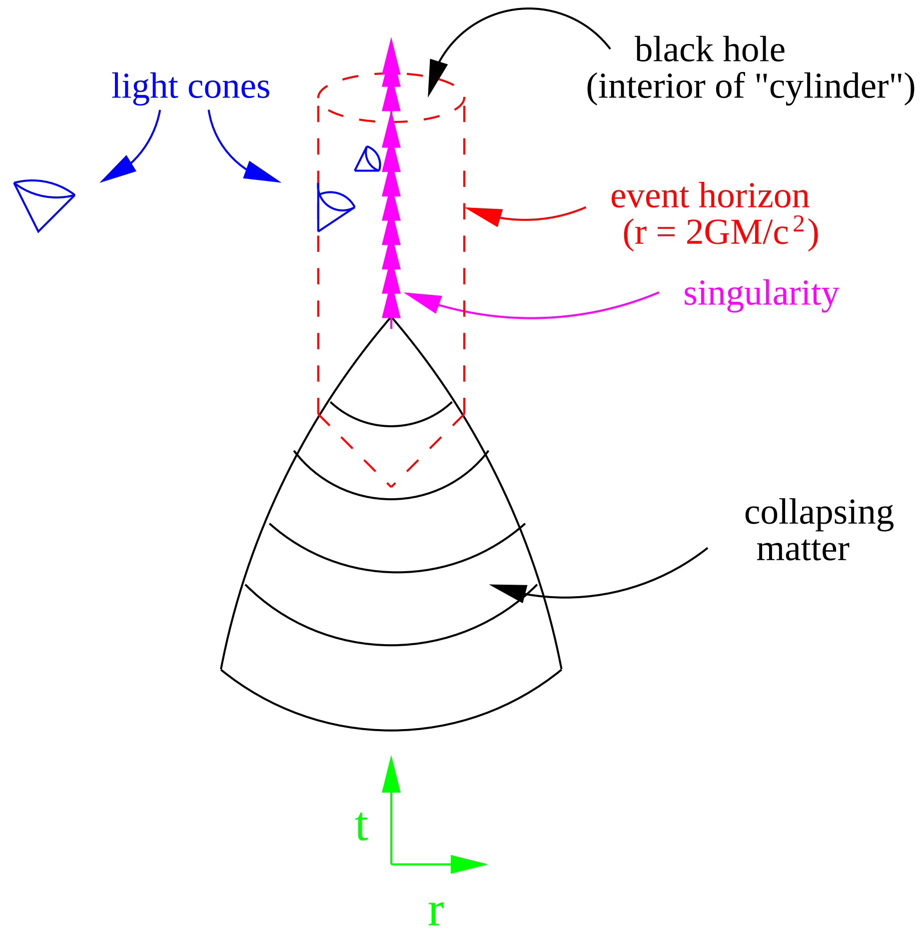
Almost all nearby galaxies show evidence for the presence of a massive black hole, and about a dozen show very strong evidence. There is convincing evidence for the presence of a black hole of mass $\sim 4 \times 10^6 M_\odot$ at the center of our own galaxy.

3. Primordial Black Holes: Could have been produced by the collapse of regions of enhanced density in the very early universe.

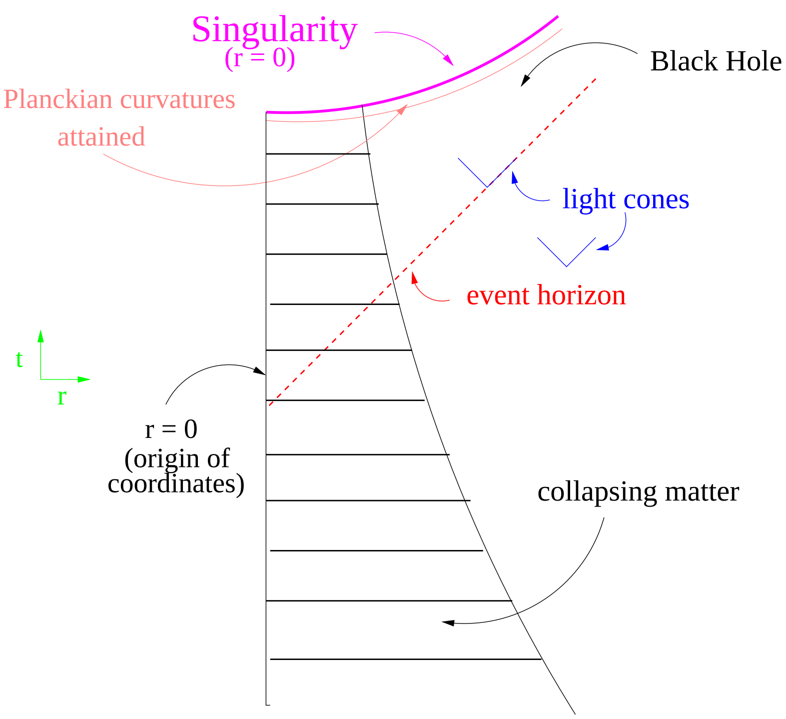
Mass range: anything

(There is no observational evidence for primordial black holes.)

Spacetime Diagram of Gravitational Collapse

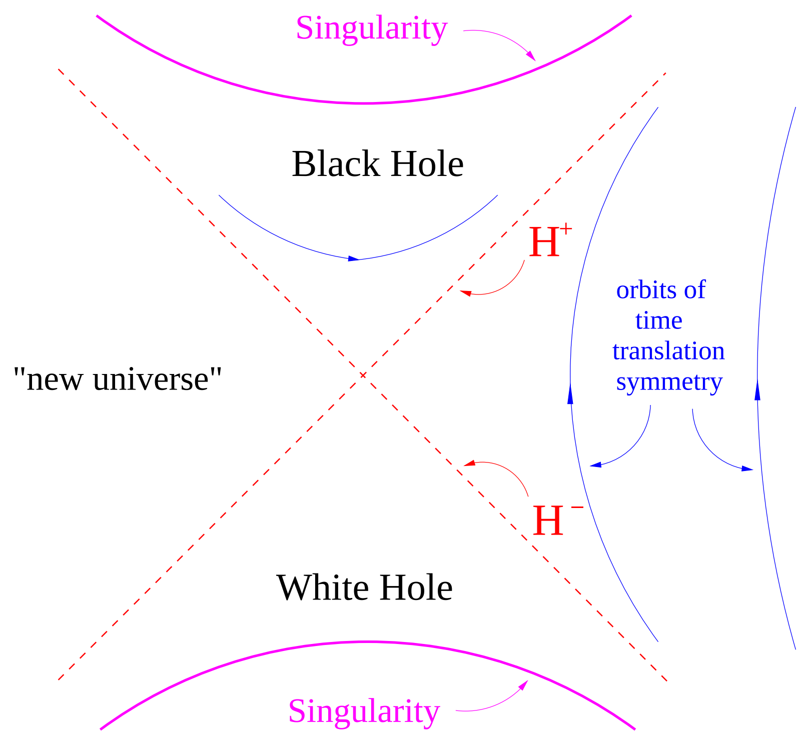


Spacetime Diagram of Gravitational Collapse with Angular Directions Suppressed and Light Cones "Straightened Out"

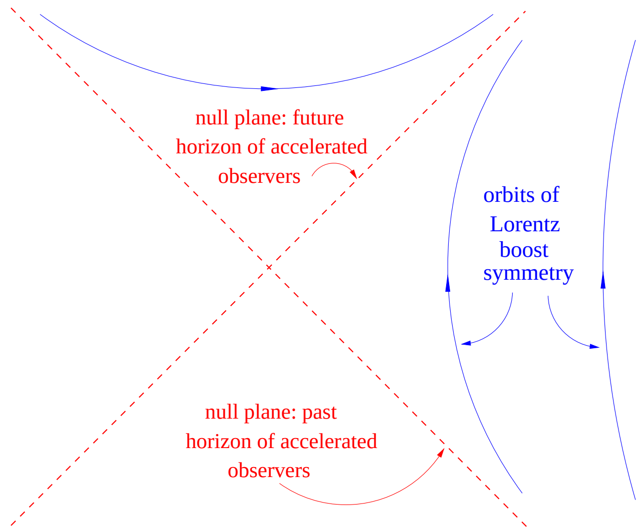


Idealized ("Analytically Continued") Black Hole

"Equilibrium State"



A Close Analog: Lorentz Boosts in Minkowski Spacetime



Note: For a black hole with $M \sim 10^9 M_\odot$, the curvature at the horizon of the black hole is smaller than the curvature in this room! An observer falling into such a black hole would hardly be able to tell from local measurements that he/she is not in Minkowski spacetime.

Black Holes and Thermodynamics

Stationary black hole $\leftrightarrow$ Body in thermal equilibrium

Consider an ordinary system composed of a large number of particles, such as the gas in a box. If one waits long enough after one has filled a box with gas, the gas will "settle down" to final state of thermal equilibrium, characterized by a small number of "state parameters", such as the total energy, E, and the total volume, V. Similarly, if one forms a black hole by gravitational collapse, it is expected that the black hole will quickly "settle down" to a stationary final state. This final state is uniquely characterized by by its total mass, M, total angular momentum, J, and total electric charge, Q.

0th Law

Thermodynamics: The temperature, T, is constant over a body in thermal equilibrium.

Black holes: The surface gravity, κ, is constant over the horizon of a stationary black hole. (κ is the limit as one approaches the horizon of the acceleration needed to remain stationary times the "redshift factor".)

1st Law

Thermodynamics:

$$\delta E = T\delta S - P\delta V$$

Black holes:

$$\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega_H \delta J + \Phi_H \delta Q$$

2nd Law

Thermodynamics:

$$\delta S \geq 0$$

Black holes:

$$\delta A \geq 0$$

Analogous Quantities

$M \leftrightarrow E \leftarrow$ But M really is E!

$\frac{1}{2\pi}\kappa \leftrightarrow T$

$\frac{1}{4}A \leftrightarrow S$

Particle Creation by Black Holes

Black holes are perfect black bodies! As a result of particle creation effects in quantum field theory, a distant observer will see an exactly thermal flux of all species of particles appearing to emanate from the black hole. The temperature of this radiation is

$$kT = \frac{\hbar\kappa}{2\pi}\,.$$

For a Schwarzshild black hole ($J = Q = 0$) we have $\kappa = c^3/4GM$, so

$$T \sim 10^{-7}\frac{M_\odot}{M}\,.$$

The mass loss of a black hole due to this process is

$$\frac{dM}{dt} \sim AT^4 \propto M^2 \frac{1}{M^4} = \frac{1}{M^2} \, .$$

Thus, an isolated black hole should "evaporate" completely in a time

$$\tau \sim 10^{73} (\frac{M}{M_\odot})^3 \mathrm{sec} \, .$$

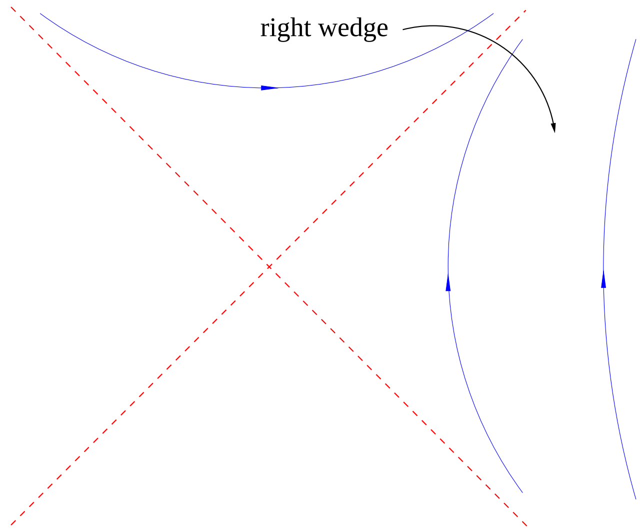
Analogous Quantities

$M \leftrightarrow E \leftarrow$ But M really is E!

$\frac{1}{2\pi}\kappa \leftrightarrow \; T \leftarrow$ But $\kappa/2\pi$ really is the (Hawking) temperature of a black hole!

$\frac{1}{4}A \leftrightarrow S$

A Closely Related Phenomenon: The Unruh Effect



View the "right wedge" of Minkowski spacetime as a spacetime in its own right, with Lorentz boosts defining a notion of "time translation symmetry". Then, when restricted to the right wedge, the ordinary Minkowski vacuum state, $|0\rangle$, is a thermal state with respect to this notion of time translations (Bisognano-Wichmann theorem). A uniformly accelerating observer "feels himself" to be in a thermal bath at temperature

$$kT = \frac{\hbar a}{2\pi c}$$

(i.e., in SI units, $T \sim 10^{-23} a$).

The Generalized Second Law

Ordinary 2nd law: $\delta S \geq 0$

Classical black hole area theorem: $\delta A \geq 0$

However, when a black hole is present, it really is physically meaningful to consider only the matter outside the black hole. But then, can decrease S by dropping matter into the black hole. So, can get $\delta S < 0$.

Although classically A never decreases, it *does* decrease during the quantum particle creation process. So, can get $\delta A < 0$.

However, as first suggested by Bekenstein, perhaps have

$$\delta S' \geq 0$$

where

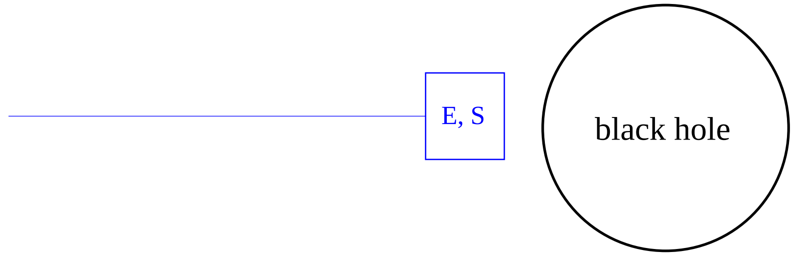
$$S' \equiv S + \frac{1}{4}\frac{c^3}{G\hbar}A$$

where S = entropy of matter outside black holes and A = black hole area.

A careful analysis of gedanken experiments strongly suggests that the generalized 2nd law is valid!

Can the Generalized 2nd Law be Violated?

Slowly lower a box with (locally measured) energy E and entropy S into a black hole.



Lose entropy S

Gain black hole entropy $\delta(\frac{1}{4}A) = \frac{\mathcal{E}}{T_{\rm b.h.}}$

But, classically, $\mathcal{E} = \chi E \to 0$ as the "dropping point" approaches the horizon, where χ is the redshift factor. Thus, apparently can get $\delta S' = -S + \delta(\frac{1}{4}A) < 0$.

However: The temperature of the “acceleration radiation” felt by the box varies as

$$T_{\text{loc}} = \frac{T_{\text{b.h.}}}{\chi} = \frac{\kappa}{2\pi\chi}$$

and this gives rise to a “buoyancy force” which produces a quantum correction to $\mathcal{E}$ that is precisely sufficient to prevent a violation of the generalized 2nd law!

Analogous Quantities

$M \leftrightarrow E \leftarrow$ But M really is E!

$\frac{1}{2\pi}\kappa \leftrightarrow T \leftarrow$ But $\kappa/2\pi$ really is the (Hawking) temperature of a black hole!

$\frac{1}{4}A \leftrightarrow S \leftarrow$ Apparent validity of the generalized 2nd law strongly suggests that $A/4$ really is the physical entropy of a black hole!

Conclusions

The study of black holes has led to the discovery of a remarkable and deep connection between gravitation, quantum theory, and thermodynamics. It is my hope and expectation that further investigations of black holes will lead to additional fundamental insights.